

RIDGE REGRESSION PROCESSING

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SUMMARY

Current navigation requirements depend on a geometric dilution of precision (GDOP) criterion. As long as the GDOP stays below a specified value, navigation requirements are met. The GDOP will exceed the specified value when the measurement geometry becomes too collinear. A new signal processing technique, called Ridge Regression Processing, can reduce the effects of nearly collinear measurement geometry; thereby reducing the inflation of the measurement errors. It is shown that the Ridge signal processor gives a consistently better mean squared error (MSE) in position than the Ordinary Least Mean Squares (OLS) estimator. The applicability of this technique is currently being investigated to improve the following areas: receiver autonomous integrity monitoring (RAIM), coverage requirements, availability requirements, and precision approaches.

BACKGROUND

Ridge Regression was developed by statisticians Hoerl and Kennard in 1970 (ref. 1). Throughout the 70's Ridge Regression was a controversial topic in the statistics world as shown by Efron (ref. 2). This was because Ridge Regression is a biased estimation technique - it goes against the traditional unbiased Ordinary Least Mean Squares (OLS) technique which has been used since Gauss. Based on the statistician's Ridge Regression techniques, a Ridge signal processor was developed in 1988 by Kelly (ref. 3) for navigation applications.

A navigation system gives a number of measurements that one tries to find system states (i.e. position, velocity, etc.) from by using a linear measurement model. The linear measurement model is given by the following equation:

$$\underline{Y} = H\underline{\beta} + \underline{e}$$

where:

\underline{Y} is the measurement vector

H is the data matrix

$\underline{\beta}$ is the system state vector

\underline{e} is the measurement noise vector

If no noise were present in the measurements, $\underline{\beta}$ could be solved for directly by taking the inverse of H and multiplying it by \underline{Y} . But since measurement noise exists, $\underline{\beta}$ must be estimated.

The OLS solution is given by the following equation:

$$\underline{\beta}_{OLS} = [H^TH]^{-1} H^T \underline{Y}$$

The GDOP, which is a factor that relates range errors and position errors, may be calculated by taking the TRACE of the first term of this equation, $(H^TH)^{-1}$. Looking more closely at this first term, one can see that when the values of the diagonal terms of the H^TH matrix get smaller, GDOP increases. The second term of the equation, $H^T \underline{Y}$, is known as the smoothing function.

The Ridge Regression solution is given below:

$$\underline{\beta}_R = [H^TH + \kappa I]^{-1} H^T \underline{Y}$$

Again, the inflation of the position errors is calculated by taking the TRACE of the first term of the equation. But notice that it now includes a variable, κ , which is added to the diagonal elements of the H^TH matrix. This variable, κ , limits the minimum values that the diagonal elements of this matrix can obtain, thereby limiting the inflation of the position errors to some maximum value. By limiting the inflation of the position errors, a better position estimate may be obtained.

RIDGE REGRESSION PROCESSING CONCEPTS

For navigation, the most important factor is the mean squared error (MSE) in position. The MSE grows as the GDOP grows. In order to understand GDOP, a look must be taken at the so-called "geometry problem". For example, if two range

measurements have a crossing angle of 90 degrees, the geometry is said to be "good"; but if the crossing angle between two range measurements is small, the geometry is said to be "bad". This geometry problem is embodied in the GDOP factor which, when using the OLS technique, is as low as 1.4 for good geometry (crossing angle is 90 degrees) and as high as 86 for bad geometry (crossing angle is 1 degree).

When bad geometry exists, the OLS technique inflates the variance of the errors around the true solution in the form of an error ellipsoid. The Ridge Regression technique adds a small bias to the true solution which in turn reduces the variance greatly, thereby giving a smaller MSE. Recall that the MSE is given by the bias term squared plus the variance, see figure 1.

A simulation is given for the Distance Measurement Equipment (DME) system with an aircraft, traveling at constant velocity, making range measurements to two DME stations. This simulation compares the performance of the OLS batch estimator to the Ridge Batch estimator. The simulation set-up is shown in figure 2. From figure 2, γ is defined to be the crossing angle between two measurements. Notice that the error ellipsoid around the aircraft is a strong function of γ (along with the measurement noise).

The simulation results are given in figure 3. Figure 3 compares the OLS estimator to the Ridge estimator for a crossing angle, γ , of 1 degree. This figure shows graphically how the OLS estimator is centered on the true solution but allows the variance of the errors to grow greatly for small crossing angles. The Ridge estimator reduces this error variance, while introducing a small bias, giving an area of errors which is off from the true solution but having an overall smaller MSE (by a factor of 100!). The reduction of the MSE is achieved by choosing an appropriate bias term, κ . (Note that when κ is equal to 0, the Ridge estimator becomes the OLS estimator).

FUTURE RESEARCH

The effectiveness of receiver autonomous integrity monitoring (RAIM) for the Global Positioning (GPS) should be significantly improved by using the Ridge Regression Processing technique (ref. 4). Navigation system integrity is defined as the detection of a signal failure and a warning to the pilot that the system is not operating within required performance limits. (The FAA requires that the pilot be notified within 10 seconds during a nonprecision approach of a signal failure for a system to have integrity.) Right now, because there are times when bad geometry

exists among the GPS satellite sub-sets, integrity monitoring cannot be performed about 15% of the time. With Ridge Regression Processing used in place of the current OLS solution, GPS monitoring performance should improve.

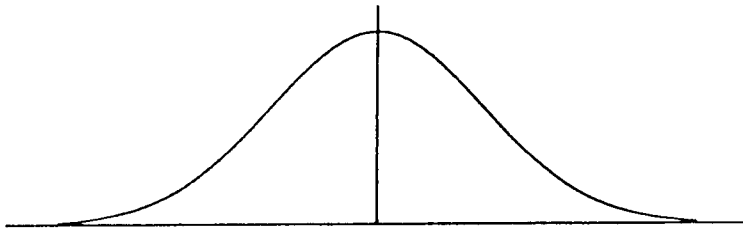
Another avenue that needs to be pursued is the implementation of a recursive Ridge processor for precision approach applications. For precision approaches, very accurate positioning and fast output of the guidance information to the pilot is needed. The current Ridge batch processor used in the above DME simulation may give better position estimates when used in cooperation with the Kalman filter. But being a batch process, it may introduce too much lag in the guidance information. Therefore, the implementation of a recursive Ridge processor is desirable.

REFERENCES

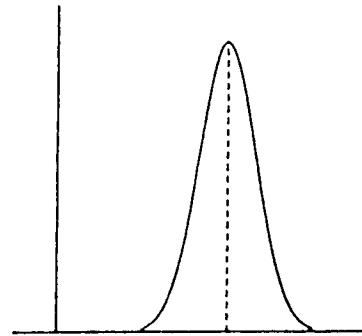
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LEAST MEAN SQUARES

RIDGE REGRESSION



UNBIASED ESTIMATION



BIASED ESTIMATION

$$\text{MSE} = \text{BIAS}^2 + \text{VARIANCE}$$

Figure 1. Comparison of unbiased and biased solutions and definition of Mean Squared Error (MSE).

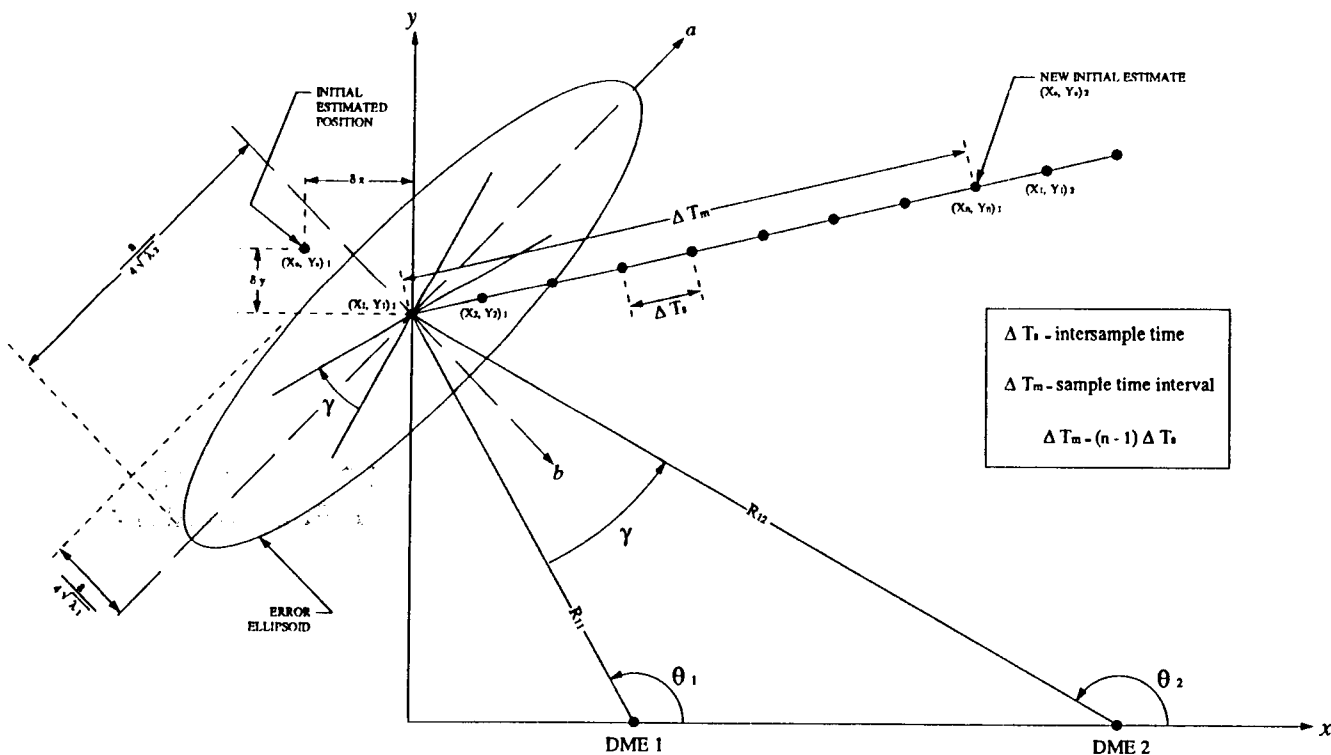


Figure 2. DME simulation geometry with an aircraft making range measurements to two DME stations.

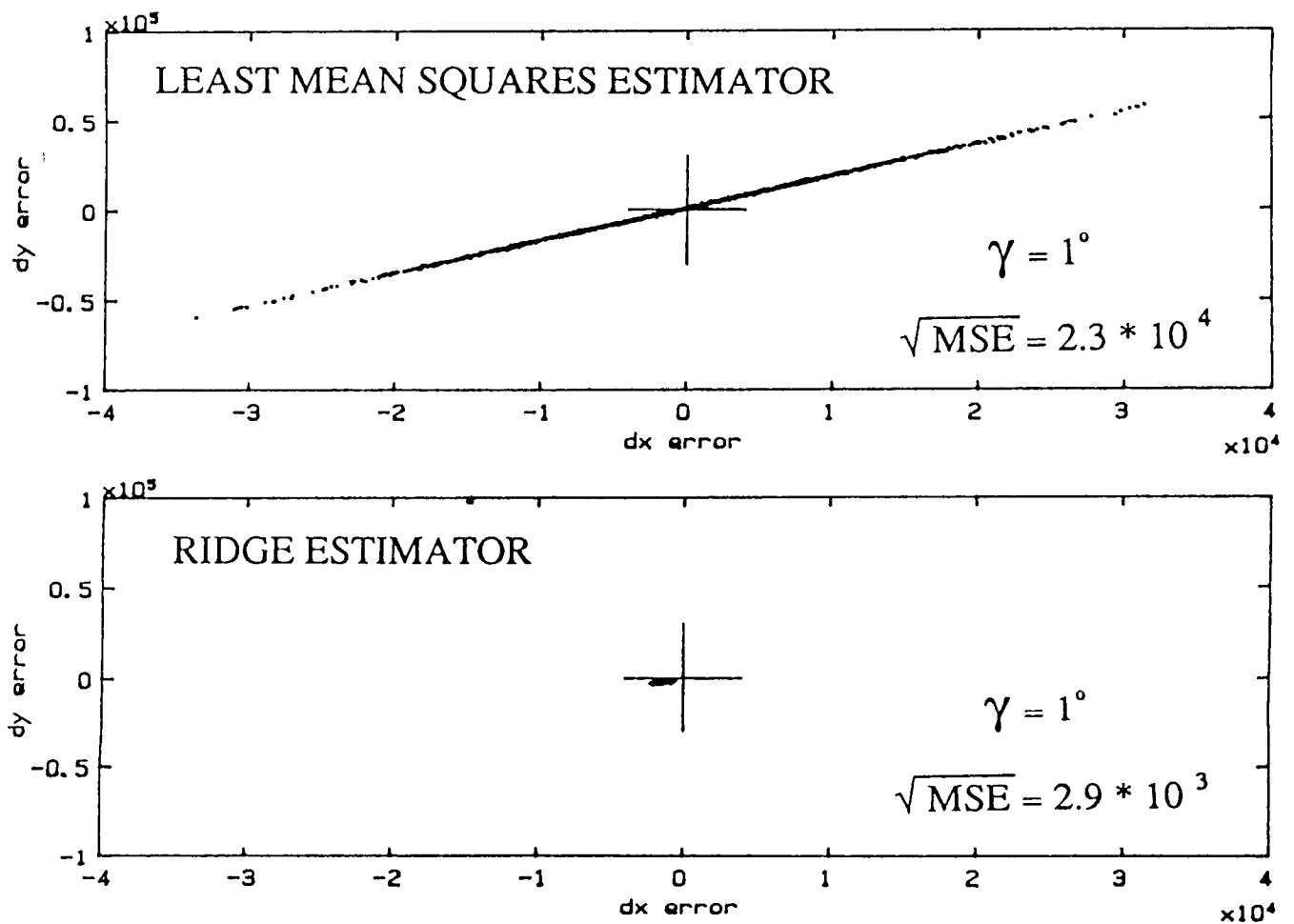


Figure 3. Comparison of the OLS estimator and the Ridge estimator through the DME simulation with the two range measurements having a small crossing angle (γ).

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